

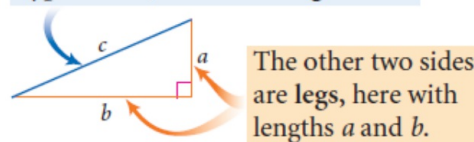
IWBAT understand the Pythagorean Theorem more deeply.

## The Pythagorean Theorem

9/22/16

Recall the parts of a triangle.

In a right triangle, the side opposite the right angle is called the hypotenuse, here with length  $c$ .



There is a special relationship between the lengths of the legs and the length of the hypotenuse. This relationship is known today as the Pythagorean Theorem. A theorem is a conjecture that has been proved.

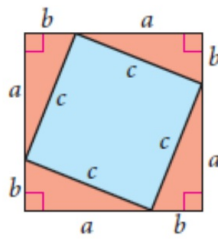
### The Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse. If  $a$  and  $b$  are the lengths of the legs, and  $c$  is the length of the hypotenuse, then  $a^2 + b^2 = c^2$ .

IWBAT understand the Pythagorean Theorem more deeply.

## The Pythagorean Theorem

You are going to perform the Investigate on DG p. 462.



### Paragraph Proof: The Pythagorean Theorem

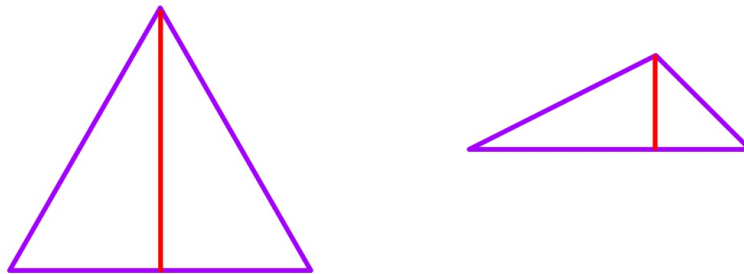
You need to show that  $a^2 + b^2$  equals  $c^2$  for the right triangles in the figure at left. The area of the entire square is  $(a + b)^2$  or  $a^2 + 2ab + b^2$ . The area of any triangle is  $(\frac{1}{2})ab$ , so the sum of the areas of the four triangles is  $2ab$ . The area of the quadrilateral in the center is  $(a^2 + 2ab + b^2) - 2ab$ , or  $a^2 + b^2$ .

If the quadrilateral in the center is a square then its area also equals  $c^2$ . You now need to show that it is a square. You know that all the sides have length  $c$ , but you also need to show that the angles are right angles. The two acute angles in the right triangle, along with any angle of the quadrilateral, add up to  $180^\circ$ . The acute angles in a right triangle add up to  $90^\circ$ . Therefore the quadrilateral angle measures  $90^\circ$  and the quadrilateral is a square. If it is a square with side length  $c$ , then its area is  $c^2$ . So,  $a^2 + b^2 = c^2$ , which proves the Pythagorean Theorem. ■

IWBAT understand the Pythagorean Theorem more deeply.

## The Pythagorean Theorem

How can we prove that the Pythagorean Theorem doesn't apply to all triangles?



IWBAT understand the Pythagorean Theorem more deeply.

## The Pythagorean Theorem

Exercises DG p. 465 #2-16 evens

IWBAT understand the Pythagorean Theorem more deeply.

## The Converse of the Pythagorean Theorem

9/26/16

IWBAT discover the converse of the  
Pythagorean Theorem

## The Converse of the Pythagorean Theorem

9/26/16

Three positive integers that work in the Pythagorean equation are called **Pythagorean triples**. For example, 8-15-17 is a Pythagorean triple because  $8^2 + 15^2 = 17^2$ .

Here are nine sets of Pythagorean triples.

3-4-5	5-12-13	7-24-25	8-15-17
6-8-10	10-24-26		16-30-34
9-12-15			
12-16-20			

- Choose any of the black triples to measure out in centimeters on your paper as line segments. Construct a triangle from this set of segments. What is the largest angle?
- Repeat with another triple.
- State your results as a conjecture.

IWBAT discover the converse of the Pythagorean Theorem

## The Converse of the Pythagorean Theorem

Converse of the Pythagorean Theorem

If the lengths of the three sides of a triangle satisfy the Pythagorean equation, then the triangle is a right triangle

IWBAT discover the converse of the Pythagorean Theorem



## The Converse of the Pythagorean Theorem

**Proof:** Converse of the Pythagorean Theorem

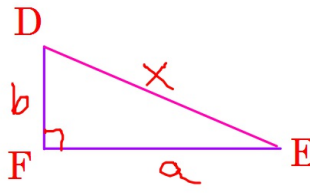
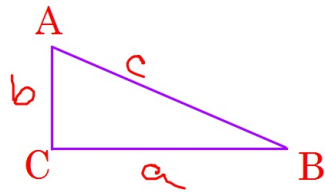
**Conjecture:** If the lengths of the three sides of a triangle work in the Pythagorean equation, then the triangle is a right triangle.

**Given:**  $a$ ,  $b$ ,  $c$  are the lengths of the sides of  $\triangle ABC$  and  $a^2 + b^2 = c^2$

**Show:**  $\triangle ABC$  is a right triangle

**Plan:** Begin by constructing a second triangle, right triangle  $DEF$  (with  $\angle F$  a right angle), with legs of lengths  $a$  and  $b$  and hypotenuse of length  $x$ . The plan is to show that  $x = c$ , so that the triangles are congruent. Then show that  $\angle C$  and  $\angle F$  are congruent. Once you show that  $\angle C$  is a right angle, then  $\triangle ABC$  is a right triangle and the proof is complete.

Critique this plan.



IWBAT discover the converse of the Pythagorean Theorem

## The Converse of the Pythagorean Theorem

Exercises DG pp. 470-471 #2-14 evens

IWBAT discover the converse of the Pythagorean Theorem

IWBAT simplify and multiply square roots and radical expressions.

Simplify  $\sqrt{50}$

Handwritten work:  $\sqrt{50}$  is broken down into  $\sqrt{25 \cdot 2}$ , with 25 and 2 written above the radical. The 25 is circled, and two 5s are written below it, indicating  $5 \cdot 5$ .

Handwritten answer:  $5\sqrt{2}$  is boxed in red.

Simplify  $\sqrt{76}$

Handwritten work:  $\sqrt{76}$  is broken down into  $\sqrt{4 \cdot 19}$ , with 4 and 19 written above the radical. The 4 is circled, and two 2s are written below it, indicating  $2 \cdot 2$ .

Handwritten work: Prime factorization of 76 is shown as  $2 \overline{) 76}$  with a quotient of 38 and a remainder of 0. Below this,  $2 \sqrt{19}$  is written.

Handwritten work: Prime factorization of 152 is shown as  $\sqrt{152}$  broken down into  $\sqrt{2 \cdot 76}$ , then  $\sqrt{2 \cdot 2 \cdot 38}$ , and finally  $\sqrt{2 \cdot 2 \cdot 2 \cdot 19}$ . The 2s are circled, indicating  $2 \cdot 2 \cdot 2$ .

Handwritten work: Prime factorization of 304 is shown as  $\sqrt{304}$  broken down into  $\sqrt{2 \cdot 152}$ , then  $\sqrt{2 \cdot 2 \cdot 76}$ , then  $\sqrt{2 \cdot 2 \cdot 2 \cdot 38}$ , and finally  $\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 19}$ . The 2s are circled, indicating  $2 \cdot 2 \cdot 2 \cdot 2$ .

IWBAT simplify and multiply square roots and radical expressions.

## Radical Expressions

Multiply  $3\sqrt{6} \times 5\sqrt{2} = 15\sqrt{12} = 30\sqrt{3}$

Handwritten work shows a tree diagram for 12:  $12 = 4 \cdot 3 = (2 \cdot 2) \cdot 3$ . Arrows indicate the simplification from  $\sqrt{12}$  to  $\sqrt{4 \cdot 3}$  and then to  $2\sqrt{3}$ .

Multiply  $7\sqrt{3} \times 2\sqrt{5} = 14\sqrt{15}$

Handwritten work shows a tree diagram for 15:  $15 = 3 \cdot 5$ . Arrows indicate the simplification from  $\sqrt{15}$  to  $\sqrt{3 \cdot 5}$ .

IWBAT simplify and multiply square roots and radical expressions.

## Radical Expressions

Exercises DG p. 474 #1, 3, 5, 6, 12, 18

Handwritten work for  $\sqrt{720}$  shows a tree diagram:  $720 = 72 \cdot 10 = (36 \cdot 2) \cdot (2 \cdot 5) = (6 \cdot 6) \cdot 2 \cdot 2 \cdot 5$ . The final simplified form is  $12\sqrt{5}$ .

Handwritten work for  $\sqrt{8200}$  shows a tree diagram:  $8200 = 82 \cdot 100 = (41 \cdot 2) \cdot (10 \cdot 10) = 2 \cdot 41 \cdot 10 \cdot 10$ . The final simplified form is  $10\sqrt{82}$ .

Handwritten work for  $\sqrt{2}$  shows a tree diagram:  $2 = 1 \cdot 2$ . The final simplified form is  $\sqrt{2}$ .

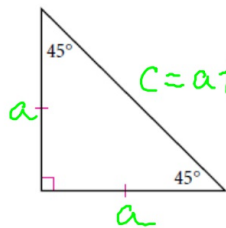
IWBAT simplify and multiply square roots and radical expressions.

## IWBAT

- discover the relationships among the lengths of the sides of a 45-45-90 triangle and a 30-60-90 triangle, and
- practice simplifying square roots.

## Two Special Right Triangles

9/28/16



What kind of triangle is this? *isosceles right Δ*

What is the relationship between the legs and the hypotenuse?

$$a^2 + a^2 = c^2$$

$$\sqrt{2a^2} = \sqrt{c^2}$$

$$a\sqrt{2} = c$$

$$\text{if } a = 38, c = 38\sqrt{2}$$

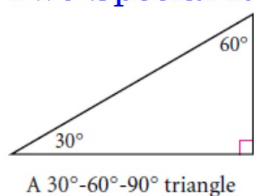
$$\text{if } c = 12, a = \frac{12}{\sqrt{2}} \left( \frac{\sqrt{2}}{\sqrt{2}} \right) = \frac{12\sqrt{2}}{2} = \underline{6\sqrt{2}}$$

## Isosceles Right Triangle Conjecture

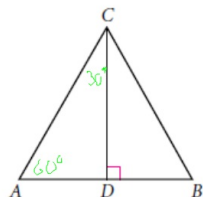
In an isosceles right triangle, if the legs have length  $l$ , then the hypotenuse has length  $\underline{l\sqrt{2}}$ .

IWBAT discover the relationships among the lengths of the sides of a 45-45-90 triangle and a 30-60-90 triangle, and practice simplifying square roots.

## Two Special Right Triangles

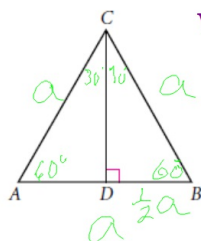


If you fold an equilateral triangle along one of its altitudes, the triangles you get are 30°-60°-90° triangles. A 30°-60°-90° triangle is half an equilateral triangle, so it also shows up often in mathematics and engineering.



IWBAT discover the relationships among the lengths of the sides of a 45-45-90 triangle and a 30-60-90 triangle, and practice simplifying square roots.

## Two Special Right Triangles

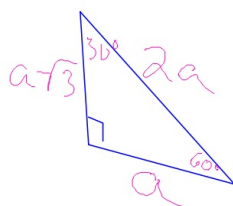


What do we know about this triangle?



$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + b^2 &= (2a)^2 = (2a)(2a) \\ a^2 + b^2 &= 4a^2 \\ -a^2 & \quad -a^2 \\ \hline b^2 &= 3a^2 \\ b &= a\sqrt{3} \end{aligned}$$

What is the relationship between the legs and the hypotenuse?



IWBAT discover the relationships among the lengths of the sides of a 45-45-90 triangle and a 30-60-90 triangle, and practice simplifying square roots.



## Two Special Right Triangles

### 30°-60°-90° Triangle Conjecture

In a 30°-60°-90° triangle, if the shorter leg has length  $a$ , then the longer leg has length  $a\sqrt{3}$  and the hypotenuse has length  $2a$ .

Use the Pythagorean Theorem to prove that this relationship is true for any 30°-60°-90° triangle.

$$a^2 + (a\sqrt{3})^2 = (2a)^2$$

$$a^2 + 3a^2 = 4a^2$$

$$4a^2 = 4a^2$$

IWBAT discover the relationships among the lengths of the sides of a 45-45-90 triangle and a 30-60-90 triangle, and practice simplifying square roots.

## Two Special Right Triangles

Exercises DG p. 477-478 #1-11 all

IWBAT discover the relationships among the lengths of the sides of a 45-45-90 triangle and a 30-60-90 triangle, and practice simplifying square roots.

IWBAT apply the Pythagorean Theorem and its converse to story problems.

What is the longest stick that will fit inside a 24-by-30-by-18-inch box?

$$d^2 = 24^2 + 30^2$$

$$d^2 = 576 + 900$$

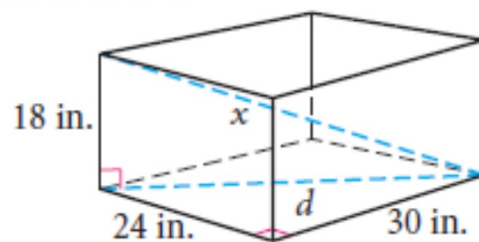
$$d^2 = 1476$$

$$x^2 = 1476 + 18^2$$

$$x^2 = 1476 + 324$$

$$x^2 = 1800$$

$$x = \sqrt{1800} = 10\sqrt{18} = 30\sqrt{2} \approx 42.4"$$



IWBAT apply the Pythagorean Theorem and its converse to story problems.

## Story Problems

Exercises DG p. 482-483 #1-5 all

IWBAT apply the Pythagorean Theorem and its converse to story problems.

## Distance in Coordinate Geometry

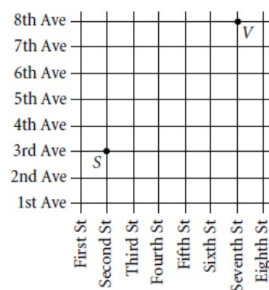
10/03/16

IWBAT discover the Pythagorean relationship on a coordinate plane, derive the equation of a circle from the distance formula, and use the distance formula to solve problems.

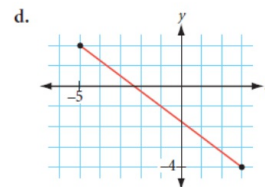
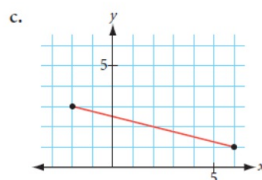
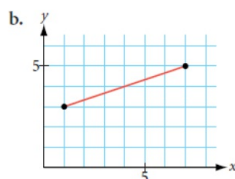
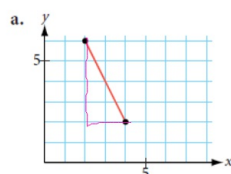
## Distance in Coordinate Geometry

10/03/16

How far is it from S to V?



With a partner, perform Investigation 1 on pp. 486-487.



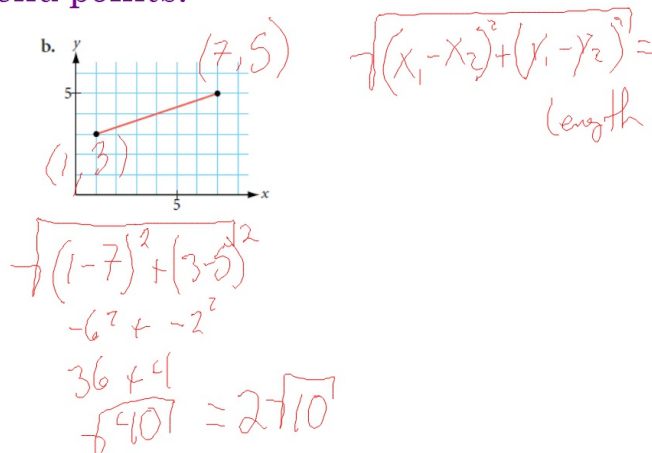
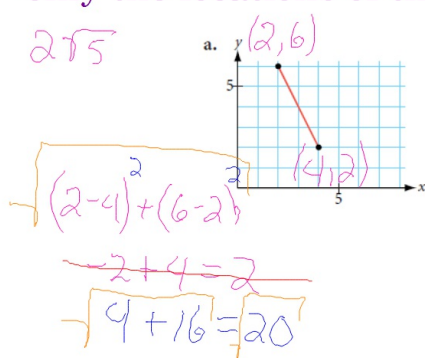
IWBAT discover the Pythagorean relationship on a coordinate plane, derive the equation of a circle from the distance formula, and use the distance formula to solve problems.

## Distance in Coordinate Geometry

What if the points are too far apart to plot? How will you find the distance between them then?

use the coordinates and subtract them  
 $(-28, -78)(642, 94)$   $\sqrt{(-28-642)^2 + (-78-94)^2}$

Find the distances between the points in a. and b. using only the locations of the end points.



IWBAT discover the Pythagorean relationship on a coordinate plane, derive the equation of a circle from the distance formula, and use the distance formula to solve problems.

## Distance in Coordinate Geometry

### Distance Formula

The distance between points  $A(x_1, y_1)$  and  $B(x_2, y_2)$  is given by

$$(AB)^2 = \left( x_1 - x_2 \right)^2 + \left( y_1 - y_2 \right)^2$$

$$\text{or } AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Find the distance between  $A(8, 15)$  and  $B(-7, 23)$ .

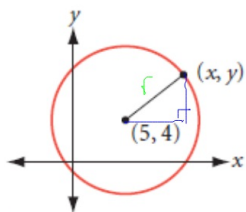
$$AB = \sqrt{(8 - (-7))^2 + (15 - 23)^2}$$

$$\sqrt{15^2 + (-8)^2} = \sqrt{225 + 64} = \sqrt{289}$$

IWBAT discover the Pythagorean relationship on a coordinate plane, derive the equation of a circle from the distance formula, and use the distance formula to solve problems.

## Distance in Coordinate Geometry

Write the equation for the circle with the center  $(5, 4)$  and radius 7.



$$r^2 = x^2 + y^2$$

$$(x-0)^2$$

$$r^2 = (x-5)^2 + (y-4)^2 \quad 49 = (x-5)^2 + (y-4)^2$$

$$r = \sqrt{(x-5)^2 + (y-4)^2} \quad 7 = \sqrt{(x-5)^2 + (y-4)^2}$$

Complete Investigate 2 on p. 488.

a. Center =  $(1, -2)$ ,  $r = 8$

b. Center =  $(0, 2)$ ,  $r = 6$

c. Center =  $(-3, -4)$ ,  $r = 10$

a.  $64 = (x-1)^2 + (y+2)^2$

b.  $36 = (x-0)^2 + (y-2)^2$

$$36 = x^2 + (y-2)^2$$

IWBAT discover the Pythagorean relationship on a coordinate plane, derive the equation of a circle from the distance formula, and use the distance formula to solve problems.



## Distance in Coordinate Geometry

### Equation of a Circle

The equation of a circle with radius  $r$  and center  $(h, k)$  is  $(x - \underline{h})^2 + (y - \underline{k})^2 = \underline{r}^2$ .

Exercises p. 489 #1-10

IWBAT discover the Pythagorean relationship on a coordinate plane, derive the equation of a circle from the distance formula, and use the distance formula to solve problems.

## Perimeter & Area via the Distance Formula

10/06/16

IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

## Perimeter & Area via the Distance Formula

### Introduction to Edgenuity

1. Retrieve a laptop & boot it up.
2. Open Firefox (not Chrome).
3. Go to the address bar and type in: learn.edgenuity.com
4. Choose Student
5. Your username is your DPS G-mail address  
(#####@dpsk12.net)
6. Enter your password (6-digit birthday)

I cannot see your password, but I can issue a temporary password so you can change it to one you can remember.

IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

## Perimeter & Area via the Distance Formula

### Geometry Semester 2

#### Geometric Modeling in Two Dimensions

- Area of Triangles and Parallelograms
- Perimeter & Area of Rhombi, Trapezoids, & Kites

Today, as a whole group we will be doing the lesson on triangles and parallelograms.

Tomorrow, you will be doing the lesson on the other three polygons individually.

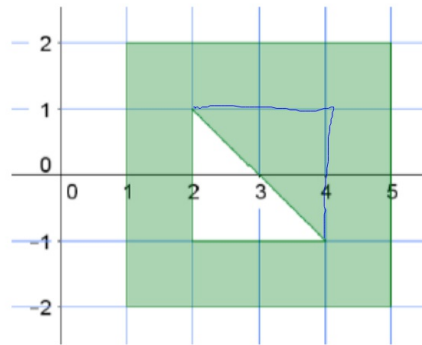
1. Warm-up
2. Instruction/Lesson
3. Summary
4. Assignment
5. Quiz

IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

## Perimeter & Area via the Distance Formula

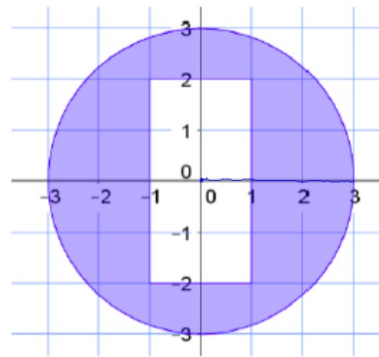
Find the area of the shaded region.

a.



$$\begin{aligned} A_{\text{square}} &= 4 \times 4 = 16 \\ - A_{\text{triangle}} &= \frac{1}{2}(2 \times 2) = 2 \\ \hline &14 \text{ sq} \end{aligned}$$

b.



$$\begin{aligned} A_{\text{circle}} &= \pi r^2 = \pi 3^2 = 28.3 \\ - A_{\text{rect}} &= 2 \times 4 = -8 \\ \hline &20.3 \text{ sq} \end{aligned}$$

IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

## Perimeter & Area via the Distance Formula

### Group 1

- Noelia
- Brisia
- Bree
- Abby
- Victoria
- Alena

### Group 2

- Sue
- Zayra
- Pam
- Tomacina
- Stef
- Erica

IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

## Perimeter & Area via the Distance Formula

Consider a triangular region in the plane with vertices  $O(0,0)$ ,  $A(5,2)$ , and  $B(3,4)$ . What is the perimeter of the triangular region? What is the area of the triangular region?

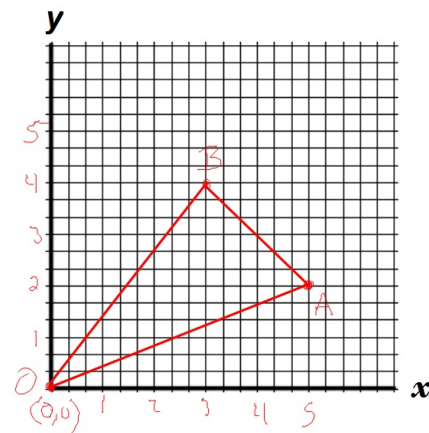
$$AO \approx \frac{2}{5} \quad BO \approx \frac{4}{3} \quad AB \approx \frac{2}{2} = 1$$

$$AO = \sqrt{(5-0)^2 + (2-0)^2} = \sqrt{5^2 + 2^2} = \sqrt{25+4} = \sqrt{29}$$

$$BO = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$AB = \sqrt{2^2 + 2^2} = \sqrt{8}$$

$$P = \sqrt{29} + 5 + \sqrt{8} \approx 13.2$$



Group 1 find the area using  $A=(b \cdot h)/2$  and an altitude.

Group 2 find the area using a different method.

IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

## Perimeter & Area via the Distance Formula

Group 1 find the area using

$A=(b \cdot h)/2$  and an altitude.

$$A = \frac{(AO \cdot h)}{2} = \frac{\sqrt{29} \cdot h}{2}$$

$$h = \sqrt{\left(3\frac{2}{3} - 3\right)^2 + \left(1\frac{1}{2} - 4\right)^2}$$

$$\sqrt{\left(\frac{2}{3}\right)^2 + \left(-2\frac{1}{2}\right)^2}$$

$$\sqrt{\frac{4}{9} + 6\frac{1}{4}}$$

$$\sqrt{\frac{16}{36} + 6\frac{9}{36}}$$

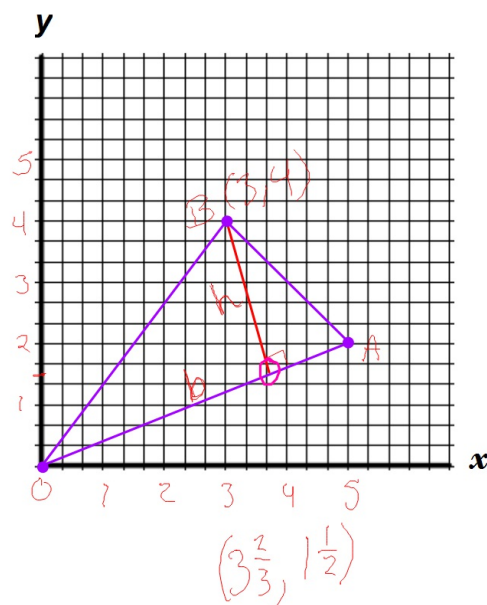
$$\sqrt{6\frac{25}{36}}$$

$$\frac{\sqrt{29} \cdot \sqrt{6\frac{25}{36}}}{2}$$

$$\frac{\sqrt{29} \cdot \sqrt{6 \cdot \frac{25}{36}}}{2}$$

$$5.49621...$$

$$\approx 5.5 \text{ u}^2$$



IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.



## Perimeter & Area via the Distance Formula

Group 2 find the area using a different method.

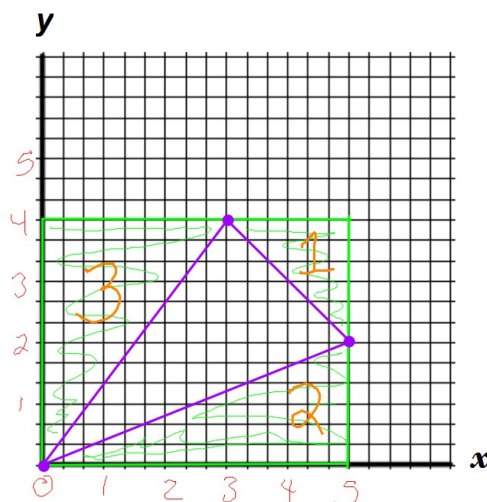
$$A_{\square} = 4 \times 5 = 20u^2$$

$$A_1 = \frac{1}{2}(2 \cdot 2) = 2u^2$$

$$A_2 = \frac{1}{2}(2 \cdot 5) = 5u^2$$

$$A_3 = \frac{1}{2}(4 \cdot 3) = 6u^2 + \frac{13u^2}{2}$$

$$\begin{array}{r} 20u^2 \\ - 13u^2 \\ \hline 7u^2 \end{array}$$

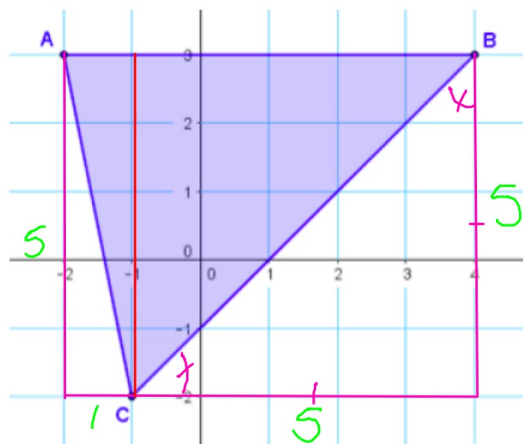


IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

## Perimeter & Area via the Distance Formula

Given the triangle below, what is its area?

What is its perimeter?



$$A_{ABC} = (6 \cdot 5) - \left[ \frac{(5 \cdot 1)}{2} + \frac{(5 \cdot 5)}{2} \right]$$

$$A_{ABC} = 30 - 15 = 15u^2$$

$$P_{ABC} = AB + BC + CA$$

$$AB = 6$$

$$BC = \sqrt{5^2 + 25^2} = \sqrt{50} = 5\sqrt{2}$$

$$CA = \sqrt{1^2 + 5^2} = \sqrt{26}$$

$$P = 6 + 5\sqrt{2} + \sqrt{26}$$

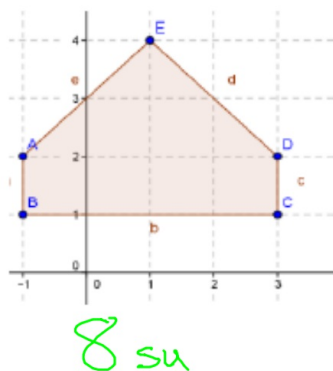
$$P \sim 46.45u \sim 18.2$$

IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.



## Perimeter & Area via the Distance Formula

Find the area of these polygons.



IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

## Perimeter & Area via the Distance Formula

Challenge: Find the area and perimeter of this polygon.

$$AE = 1$$

$$AB = \sqrt{2^2 + 2^2} = \sqrt{8}$$

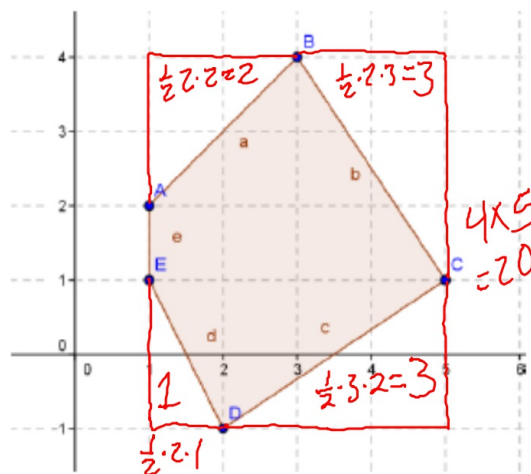
$$BC = \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$CD = \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$DE = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$P = 1 + \sqrt{8} + 2\sqrt{13} + \sqrt{5}$$

$$P \sim 13.28 u$$



IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

## Perimeter & Area via the Distance Formula

Find the perimeter and area of the polygons in exercises p.370-371 #1-3 and area of #4 & 5.  
To turn in with full header.

IWBAT calculate the perimeter and area of polygons on the coordinate plane by using the distance formula.

What should be in your notebook as of today:

Exercises DG p. 465 #2-16 evens

Exercises DG pp. 470-471 #2-14 evens

Exercises DG p. 474 #1, 3, 5, 6, 12, 18

Exercises DG p. 477-478 #1-11 all

Exercises DG p. 482-483 #1-5 all

Exercises p. 489 #1-10